

Time-varying Tracking Algorithm for Active Distribution Grid Energy Management Considering Uncertainty of Feeder Parameters

Jiexuan Wu, Mingbo Liu, Wentian Lu, Kaijun Xie, and Min Xie

Abstract—To cope with rapidly fluctuating distributed energy resources and uncontrolled loads, this paper formulates a time-varying dispatching model for distribution grids considering the uncertainty of feeder parameters, and develops a novel tracking algorithm by utilizing time-varying information of objective functions and constraints. Unlike existing methods, the proposed approach aims to build a dynamic system to obtain the evolution of the optimal trajectory, which demonstrates online tracking capability with an asymptotically vanishing error. The proposed approach contains two key terms: a prediction term tracking change in the optimal solution based on the time-varying nature of system power, and a correction term pushing the solution toward the optimum based on Newton's method. Moreover, the proposed algorithm can be applied in the absence of an accurate network model by leveraging voltage measurements to identify true voltage sensitivity parameters. Simulations for a sample distribution network are carried out to verify the approach.

Index Terms—Distribution grids, dynamic system, time-varying optimization, tracking algorithm, voltage measurements.

NOMENCLATURE

t	Index for time.
t_k, k	Indices for time instant.
i/l	Index for nodes/lines.
N	Set of nodes.
R	Set of real numbers.
V_{nom}	Voltage nominal value.
$u^{\min}/u^{\max}$	Lower/upper bounds of control variables.
P_L/Q_L	Real/reactive power loads.
A, B	Voltage sensitivity matrixes.

M	Node-to-branch incidence matrix.
ϖ_i	Feasible operating range of P_{gi} and Q_{gi} .
$P_i^{\text{av}}/Q_i^{\text{av}}$	Available real/reactive power of DER i .
θ_i	Power factor angle of DER i .
c	Barrier parameter.
s	Slack function.
P_g/Q_g	Output real/reactive power vectors of DERs.
P/Q	Net real/reactive power vectors.
V	Voltage magnitude vector.
u	Control variable vector.
P_{gi}/Q_{gi}	Real/reactive power outputs of DER i .
V_i	Voltage magnitude at node i .

I. INTRODUCTION

DISTRIBUTED energy resources (DERs) are expected to be dispersed in future distribution networks through power electronic devices to reduce carbon footprints [1]. Under influence of ambient conditions, some renewable energy sources, such as photovoltaic (PV) systems and wind turbines (WTs), show fast time-varying characteristics. For example, PV output may change dramatically in seconds when affected by clouds, causing network security and voltage quality problems [2]. Additionally, DC/AC converters and electronic modulation technologies are widely used in distributed power supplies [3], which enables them to be rapidly regulated, e.g., in 0.02 s (for 50 Hz frequency). This feature suggests DERs' fast-responding characteristics can enhance active operation and regulation ability of distribution networks if optimal determination of DERs' setpoints can match their adjustment time. Therefore, there is an urgent need for computationally frugal algorithms that can produce solutions in a timely fashion.

To keep up with rapid changes in the environment, measurement-based online methods [2], [4]–[8] have been developed on a timescale of seconds. A feedback control strategy [5] for optimal reactive power setpoints for micro-generators was proposed to provide real-time reactive power compensation. Online primal-dual-type methods [6]–[8] were applied to develop real-time feedback algorithmic frameworks for time-varying optimization problems. Shortcoming shared by the above-mentioned methods is that they only carry out “correction” steps by utilizing current information of the problem parameters but not perform a “prediction” step [9]. These approaches need iterations during a sampling interval to converge toward the optimum of the sampled time-invariant

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problem, while actual optimum drifts with time. Therefore, these approaches are likely to induce a large tracking error [10].

To reduce tracking error, discrete-time model predictive control methods [11], [12] and prediction-correction (PC) methods [13]–[17] utilized prediction information of the problem parameters to identify the change of the optimum and correct the prediction based on newly acquired information. Simonetto *et al.* [17] proposed a discrete-time PC approach to implement time-varying convex optimization and extended it to optimization of DER operation in distribution systems, while other algorithms [13]–[16] are temporarily applied to simple mathematical examples. However, discrete PC methods still cannot fully identify the dynamics within the interval, whose asymptotical error depends on length of sampling interval and iteration steps of “correction.” To track the optimal solution without errors, a continuous-time PC method was developed for an unconstrained optimization problem [18]. Fazlyab *et al.* [10] developed a PC interior-point method for time-varying convex optimization problems and applied it to solve a collision-free robot navigation problem.

For distribution grids, in addition to fast-fluctuating DERs, uncertainty of feeder parameters impedes solving the time-varying optimization problem in a timely fashion. In distribution networks, the nonlinear relationship between nodal voltages and power injections will cause the optimal dispatching model to be non-convex and difficult to solve. Approximate linear power flow models are applied instead in most online methods, e.g., [2], [5]–[7], [17]. However, these schemes assume the ideal model of distribution networks can be obtained, so they may not work properly when an accurate model is not available or if feeder parameters change (due to environmental changes). For this scenario, data-driven methods [19]–[22] are effective solutions. A least squares estimator was utilized [19] to compute sensitivity coefficients of voltage magnitude and power loss. Forward and inverse regression methods [20] and a recursive weighted least squares method [21] were studied for the case in which the model of the distribution system is not completely known. Xu *et al.* [22] proposed an approach that exploited the structural characteristics of balanced radial distribution networks to reduce the number of estimated parameters. In this paper, voltage measurements throughout the feeder are collected, and structural characteristics of distribution networks are exploited to identify true voltage sensitivity parameters.

To better track dynamics within the interval, a novel tracking algorithm for distribution grids is developed in this paper to identify time-varying optimal power setpoints of DERs with an asymptotically vanishing error. The fast-changing nature of renewable energy resources and uncontrolled loads is taken into account in the prediction term to identify change of the optimum; a correction term represented by Newton’s method pushes the solution toward the optimum. Moreover, in the absence of an exact system model, voltage sensitivity parameters are identified by measurement information. By doing so, optimal trajectories of DERs’ power setpoints and network voltages will automatically adapt to system disturbances, such

as renewable energy sources, uncontrollable loads, and system model parameters.

The main contributions of this paper are as follows:

1) A time-varying tracking algorithm with “prediction” and “correction” terms is constructed to identify optimal solutions of distribution grids with an asymptotically vanishing error. Compared with existing methods of distribution system time-varying optimization (with different degrees of tracking error), the proposed method is the first to maintain accurate tracking performance in a very short time, such as 0.02 s. Moreover, the proposed algorithm’s rapid calculation ability enables optimal determination of DERs’ setpoints to match adjustment time of DER inverters, allowing the algorithm to cope with rapid changes of renewable energy resources.

2) To avoid inaccuracy of optimization results caused by absence of an accurate network model or variation of feeder parameters due to environmental changes, voltage sensitivity parameters are identified by measurements in the optimization process. Exploiting structural characteristics of distribution networks, voltage sensitivity matrices are obtained based on a few online measurements, which incur no extra computational burden to the algorithm.

In the following sections, we will formulate the time-varying optimization problem of distribution grids with DERs. Next, a novel tracking algorithm based on voltage measurements is proposed. Section III presents simulation results on a sample system. Finally, the conclusion is provided.

II. PROBLEM FORMULATION

A distribution feeder is considered with L distribution lines and $n+1$ nodes, which are collected in the set $N \cup \{0\}$, where $N := \{1, \dots, n\}$. Node 0 represents the point of common coupling (PCC). A time-varying minimization problem that captures the optimization objective and constraints is formulated as (1), which enables us to capture variability of system conditions.

$$\min_{\mathbf{P}_g, \mathbf{Q}_g} f(\mathbf{P}_{gi}, \mathbf{Q}_{gi}, t) = \sum_{i \in N} C_i(\mathbf{P}_{gi}, \mathbf{Q}_{gi}, t) + \frac{\gamma}{2} \|\mathbf{V} - \mathbf{V}_{\text{nom}}\|^2 \quad (1a)$$

$$\text{s.t. } (\mathbf{P}_{gi}, \mathbf{Q}_{gi}) \in \varpi_i(t), \forall i \in N, \quad (1b)$$

$$V_i^{\min} \leq V_i(\mathbf{P}(t), \mathbf{Q}(t)) \leq V_i^{\max}, \forall i \in N, \quad (1c)$$

where $C_i(\mathbf{P}_{gi}, \mathbf{Q}_{gi}, t) = C_{pi}(\mathbf{P}_{gi} - \mathbf{P}_i^{\text{tar}}(t))^2 + C_{qi}\mathbf{Q}_{gi}^2$ represents the cost objective associated with DERs; C_{pi} and C_{qi} are cost coefficients of real and reactive power of DER i , respectively; $\mathbf{P}_g = [P_{g1}; P_{g2}; \dots; P_{gn}]$ and $\mathbf{Q}_g = [Q_{g1}; Q_{g2}; \dots; Q_{gn}]$; γ is a positive constant that is used to make a trade-off between power cost of DERs and voltage deviation of the network, which can be set to 1 without loss of generality [7]; $\mathbf{V}_{\text{nom}}$ denotes voltage nominal value; and $\varpi_i(t)$ represents feasible operating range of DER i . Function $V_i(\mathbf{P}(t), \mathbf{Q}(t))$ captures the relationship between voltage magnitude V_i at node i and net real and reactive power $\mathbf{P}(t), \mathbf{Q}(t)$.

Specifically, for PV systems and WTs, $\mathbf{P}_i^{\text{tar}}(t) = \mathbf{P}_i^{\text{av}}(t)$ in (1a), with $\mathbf{P}_i^{\text{av}}(t)$ denoting maximum available real power of PV or WT at node i . Set ϖ_i is given by:

$$\varpi_i(t) = \left\{ \begin{array}{l} 0 \leq P_{gi} \leq P_i^{\text{av}}(t), |Q_{gi}| \leq Q_i^{\text{av}}(t), \\ Q_i^{\text{av}}(t) = P_i^{\text{av}}(t) \tan \theta_i \end{array} \right\}$$

where $Q_i^{\text{av}}(t)$ represents reactive power limitation of DER i ; θ_i is power factor angle.

As for energy storage systems (ESSs), $P_i^{\text{tar}}(t) = 0$ in (1a), and set ϖ_i is given by:

$$\varpi_i(t) = \begin{cases} -P_{i,\max}^{\text{ch}} \leq P_{gi} \leq P_{i,\max}^{\text{dis}} \\ -\frac{1}{\eta_c}(W_{i,\max} - W_{i,0}) \leq P_{gi} \leq \eta_d(W_{i,0} - W_{i,\min}) \end{cases}$$

where $P_{i,\max}^{\text{ch}}/P_{i,\max}^{\text{dis}}$ denotes rated charging/discharging power of the ESS i ; η_c/η_d are charging/discharging efficiency; and $W_{i,\min}/W_{i,\max}$ denote the lower/upper bound of total energy storage of an ESS. $W_{i,0}$ represents current energy storage of an ESS, which can be calculated from information at the previous moment.

Let $\mathbf{P}(t) = \mathbf{P}_g + \mathbf{P}_L(t)$ and $\mathbf{Q}(t) = \mathbf{Q}_g + \mathbf{Q}_L(t)$. The nonlinear relationship $\mathbf{V}(\mathbf{P}(t), \mathbf{Q}(t))$ can be approximated by a linear model [14] as follows:

$$\mathbf{V} = \mathbf{A}\mathbf{P}(t) + \mathbf{B}\mathbf{Q}(t) + \mathbf{c} \quad (2)$$

where $\mathbf{A} \in \mathbb{R}^{n \times n}$ and $\mathbf{B} \in \mathbb{R}^{n \times n}$ are time-varying voltage sensitivities (see, e.g., [22]) that can be estimated from a few measurements using a data-driven algorithm in this paper. We will describe in detail in Section III how to estimate these time-varying parameters.

Let $\mathbf{u} = [\mathbf{P}_g; \mathbf{Q}_g]$ be the control variables of problem (1), $\mathbf{D} = [\mathbf{A}\mathbf{B}]$, and $\mathbf{W}(t) = [\mathbf{P}_L(t); \mathbf{Q}_L(t)]$ capture the uncontrollable load. Then, problem (1) can be rearranged in the following quadratic form:

$$\min_{\mathbf{u}} f(\mathbf{u}, t) = \mathbf{u}^T \frac{\mathbf{K}}{2} \mathbf{u} + \mathbf{d}(t)^T \mathbf{u} + \frac{\gamma}{2} \|\mathbf{V} - \mathbf{V}_{\text{nom}}\|^2 \quad (3a)$$

$$\text{s.t. } \mathbf{u}^{\min}(t) \leq \mathbf{u} \leq \mathbf{u}^{\max}(t) \quad (3b)$$

$$\mathbf{V}^{\min} \leq \mathbf{V} = \mathbf{c} + \mathbf{D}\mathbf{u} + \mathbf{D}\mathbf{W}(t) \leq \mathbf{V}^{\max} \quad (3c)$$

where $\mathbf{K} = \text{diag}(2C_{pi}, 2C_{qi}, i \in N)$; $\mathbf{d}(t) = [-2C_{pi}P_i^{\text{tar}}(t), 0]^T$, $i \in N$ and $\mathbf{0}$ is a $(1 \times n)$ -dimensional vector, all entries in which are 0.

III. PROPOSED METHOD

A. Prediction-Correction Tracking Algorithm

Let $\mathbf{u}^*(t)$ be the optimal solution of problem (3). Notice there are $p = 2n + n$ time-varying inequality constraint functions included in (3b) and (3c), which can be compactly written as $f_i(\mathbf{u}, t) \leq 0$ for $i \in \{1, 2, \dots, p\}$, or $i \in [p]$. Inspired by traditional interior-point methods [23], we used the following barrier function in (4) to deal with constrained problem (3):

$$\Phi(\mathbf{u}, s, c, t) = f(\mathbf{u}, t) - \frac{1}{c(t)} \sum_{i=1}^p \ln(s(t) - f_i(\mathbf{u}, t)) \quad (4)$$

where barrier parameter $c(t)$ is an increasing function satisfying $\lim_{t \rightarrow \infty} c(t) = \infty$ and slack $s(t)$ is a decreasing function satisfying $\lim_{t \rightarrow \infty} s(t) = 0$, which is introduced to ensure $s(t) > f_i(\mathbf{u}, t)$, $i \in [p]$ holds for all times $t \geq 0$. In particular, choosing $s(0) \geq \max_{i \in [p]} \{f_i(\mathbf{u}(0), 0)\}$ is sufficient to make this case true, as verified in [10]. The barrier function then tends to value 0 when inequality constraints are satisfied,

and $+\infty$ in the opposite case. Specifically, when inequality constraints are satisfied, then item $s(t) - f_i(\mathbf{u}, t) > 0$. Because $\ln(s(t) - f_i(\mathbf{u}, t))$ is much smaller than $c(t)$, the barrier function tends to value 0. When inequality constraints are not satisfied, then item $s(t) - f_i(\mathbf{u}, t)$ tends to 0^+ . In this case, $\ln(s(t) - f_i(\mathbf{u}, t))$ tends to $-\infty$, and thus the barrier function tends to $+\infty$.

The optimal solution $\hat{\mathbf{u}}^*(t)$ of (4) should satisfy first-order optimality condition $\nabla_{\mathbf{u}} \Phi(\hat{\mathbf{u}}^*(t), s(t), c(t), t) = 0$, $\forall t \geq 0$, and thus its derivative should also satisfy formula (5):

$$\dot{\nabla}_{\mathbf{u}} \Phi = \nabla_{uu} \Phi \dot{\hat{\mathbf{u}}^*}(t) + \nabla_{us} \Phi \dot{s} + \nabla_{uc} \Phi \dot{c} + \nabla_{ut} \Phi = 0 \quad (5)$$

where $\nabla_{\mathbf{u}} \Phi$ represents total derivative of $\nabla_{\mathbf{u}} \Phi$ to t , while $\nabla_{us} \Phi$, $\nabla_{uc} \Phi$, and $\nabla_{ut} \Phi$ represent the partial derivative of $\nabla_{\mathbf{u}} \Phi$ to s , c , and t , respectively. Solving (5) for $\dot{\hat{\mathbf{u}}^*}(t)$ we obtain

$$\dot{\hat{\mathbf{u}}^*}(t) = -\nabla_{uu}^{-1} \Phi (\nabla_{us} \Phi \dot{s} + \nabla_{uc} \Phi \dot{c} + \nabla_{ut} \Phi) \quad (6)$$

Equation (6) is called the “prediction” term, which is used to predict how the optimal solution changes over time by considering time-varying characteristics of problem parameters. However, if we cannot obtain initial optimal solution $\hat{\mathbf{u}}^*(0)$ or any optimal solution $\hat{\mathbf{u}}^*(t_0)$ for some $t_0 \geq 0$, then (6) cannot be simply relied on to track evolution of $\hat{\mathbf{u}}^*(t)$.

To push the solution to the optimum, Newton’s method is implemented on $\Phi(\mathbf{u}, s, c, t)$ to rapidly converge on its minimizer $\hat{\mathbf{u}}^*(t)$, the continuous-time version of which yields the following correction term:

$$\mathbf{u}(t) = -\nabla_{uu}^{-1} \Phi(\mathbf{u}, s, c, t) \nabla_{\mathbf{u}} \Phi(\mathbf{u}, s, c, t) \quad (7)$$

By combining (6) and (7), a complete prediction-correction dynamic system can be obtained:

$$\dot{\mathbf{u}}(t) = -\nabla_{uu}^{-1} \Phi [\Lambda \nabla_{\mathbf{u}} \Phi + \nabla_{us} \Phi \dot{s} + \nabla_{uc} \Phi \dot{c} + \nabla_{ut} \Phi] \quad (8)$$

where $\Lambda = \alpha \mathbf{I}$ for some $\alpha > 0$, and $\mathbf{I}$ denotes an identity matrix.

Notice the dynamic system (8) includes the prediction term $\nabla_{ut} \Phi$, whose computation involves terms of DERs’ available power and uncontrollable load drift over time. In online applications, limited information about these terms is available. Here, assume knowledge of the time-varying power within 1 s. Then Hermite polynomials can be applied to approximate continuous-time trajectory of data sets with the desired level of accuracy [24]. That is to say, a derivative of the DER’s available power and uncontrollable load to time is known in an ultra-short amount of time.

Specifically for problem (3), parameters in dynamic system (8) can be expressed as:

$$\nabla_{\mathbf{u}} \Phi = \mathbf{K}\mathbf{u} + \mathbf{d} + \gamma \mathbf{D}^T (\mathbf{V} - \mathbf{V}_{\text{nom}}) - (\mathbf{K}_1 + \mathbf{K}_2 + \mathbf{K}_3 + \mathbf{K}_4)/c \quad (9)$$

$$\nabla_{uu} \Phi = \mathbf{K} + \gamma \mathbf{D}^T \mathbf{D} - (\bar{\mathbf{K}}_1 + \bar{\mathbf{K}}_2 + \bar{\mathbf{K}}_3 + \bar{\mathbf{K}}_4)/c \quad (10)$$

$$\nabla_{us} \Phi = -(\mathbf{K}_{1s} + \mathbf{K}_{2s} + \mathbf{K}_{3s} + \mathbf{K}_{4s})/c \quad (11)$$

$$\nabla_{uc} \Phi = (\mathbf{K}_1 + \mathbf{K}_2 + \mathbf{K}_3 + \mathbf{K}_4)/c^2 \quad (12)$$

$$\nabla_{ut} \Phi = \nabla_t \mathbf{d} + \mathbf{D}^T \mathbf{D} \nabla_t \mathbf{W} - (\nabla_t \mathbf{K}_1 + \nabla_t \mathbf{K}_2 + \nabla_t \mathbf{K}_3 + \nabla_t \mathbf{K}_4)/c, \quad (13)$$

Let $h'_i = s - u_i^{\min} + u_i$, $h''_i = s - u_i + u_i^{\max}$, $g'_i = s - V_i^{\min} + V_i$, $g''_i = s - V_i + V_i^{\max}$, and e_i be a basis vector. Then specific parameters in (9)–(13) can be expressed as

$$\begin{cases} \mathbf{K}_1 = [1/h'_i]_{2n \times 1}, \mathbf{K}_2 = [-1/h''_i]_{2n \times 1} \\ \mathbf{K}_{1s} = [-1/(h'_i)^2]_{2n \times 1}, \mathbf{K}_{2s} = [1/(h''_i)^2]_{2n \times 1} \\ \bar{\mathbf{K}}_1 = \text{diag}(-1/(h'_i)^2), \bar{\mathbf{K}}_2 = \text{diag}(-1/(h''_i)^2) \\ \nabla_t \mathbf{K}_1 = \left[\frac{du_i^{\min}}{dt} / (h'_i)^2 \right]_{2n \times 1} \\ \nabla_t \mathbf{K}_2 = \left[\frac{du_i^{\max}}{dt} / (h''_i)^2 \right]_{2n \times 1} \\ i \in [2n] \\ \mathbf{K}_3 = \sum_{i=1}^n 1/g'_i \mathbf{D}^T e_i, \mathbf{K}_4 = \sum_{i=1}^n -1/g''_i \mathbf{D}^T e_i \\ \mathbf{K}_{3s} = \sum_{i=1}^n -1/(g'_i)^2 \mathbf{D}^T e_i, \mathbf{K}_{4s} = \sum_{i=1}^n 1/(g''_i)^2 \mathbf{D}^T e_i \\ \bar{\mathbf{K}}_3 = \sum_{i=1}^n -1/(g'_i)^2 (e_i^T \mathbf{D})^T (e_i^T \mathbf{D}) \\ \bar{\mathbf{K}}_4 = \sum_{i=1}^n -1/(g''_i)^2 (e_i^T \mathbf{D})^T (e_i^T \mathbf{D}) \\ \nabla_t \mathbf{K}_3 = \sum_{i=1}^n -1/(g'_i)^2 (e_i^T \mathbf{D})^T (e_i^T \mathbf{D}) \nabla_t \mathbf{W} \\ \nabla_t \mathbf{K}_4 = \sum_{i=1}^n -1/(g''_i)^2 (e_i^T \mathbf{D})^T (e_i^T \mathbf{D}) \nabla_t \mathbf{W} \\ i \in [n] \\ \nabla_t \mathbf{d} = [-2C_{pi} dP_i^{\text{tar}}/dt, i \in N \ 0]^T \\ \nabla_t \mathbf{W} = [\nabla_t \mathbf{P}_L^T \ \nabla_t \mathbf{Q}_L^T]^T \end{cases}$$

B. Estimation of Voltage Sensitivity Parameters

In this paper, the nonlinear relationship $\mathbf{V}(\mathbf{P}(t), \mathbf{Q}(t))$ is approximated by the linear model (2). Let $\bar{\mathbf{M}} \in R^{(n+1) \times L}$ represent the node-to-branch incidence matrix of the distribution network, and $\mathbf{M}$ be a matrix obtained from $\bar{\mathbf{M}}$ by removing the row associated with PCC. Specifically, in balanced radial distribution networks, voltage sensitivity matrices can be expressed as:

$$\mathbf{A} = (\mathbf{M}^{-1})^T \text{diag}(\bar{\mathbf{r}}) \mathbf{M}^{-1} \quad (14a)$$

$$\mathbf{B} = (\mathbf{M}^{-1})^T \text{diag}(\bar{\mathbf{x}}) \mathbf{M}^{-1} \quad (14b)$$

where $\bar{\mathbf{r}}$ and $\bar{\mathbf{x}} \in R^{L \times 1}$ whose elements represent correlation coefficients between two connected nodes, and they are equivalent to line resistance and line reactance, respectively, under no-load conditions.

Assuming that network topology configurations $\mathbf{M}$ (which would not change frequently within a short period) and resistance-to-reactance ratios ς_l of distribution lines are available, then $\bar{\mathbf{r}}$ and $\bar{\mathbf{x}}$ can be identified using only a few measurements in the design. At time t_{k+1} , assume knowledge of voltage magnitude measurements $\mathbf{V}[k]$, and power injections $\mathbf{P}[k]$ and $\mathbf{Q}[k]$, where $k \in \kappa = \{t_{k-m}, \dots, t_k\}$. Let $\mathbf{v}[k] = \mathbf{V}[k] - \mathbf{c}$, where $\mathbf{c} = \mathbf{1}_n$ is an n -dimensional column

vector with all elements as 1 [8]. $\hat{\mathbf{r}}$ and $\hat{\mathbf{x}}$ represent estimates of $\bar{\mathbf{r}}$ and $\bar{\mathbf{x}}$, respectively. The parameter estimation problem can be constructed as follows [22]:

$$(\hat{\mathbf{r}}, \hat{\mathbf{x}}) = \arg \min_{\hat{\mathbf{r}}, \hat{\mathbf{x}}} \sum_{k \in \kappa} \eta^{t_k - k} \|\mathbf{A}\mathbf{P}[k] + \mathbf{B}\mathbf{Q}[k] - \mathbf{v}[k]\|^2 \quad (15a)$$

$$\text{s.t.} \quad (14a), (14b) \quad (15b)$$

where $\|\cdot\|^2$ denotes the L_2 -norm, and $\eta \in (0, 1]$ is a forgetting factor.

Let $\chi_l[k] = \eta^{(t_k - k)/2} (\varsigma_l \mathbf{P}[k] + \mathbf{Q}[k])$, $\Upsilon_l = (\mathbf{M}^{-1})^T e_l e_l^T \mathbf{M}^{-1}$, and define

$$\mathbf{Z}[t_k] = \begin{bmatrix} \Upsilon_1 \chi_1[t_{k-m}] & \cdots & \Upsilon_L \chi_L[t_{k-m}] \\ \vdots & \ddots & \vdots \\ \Upsilon_1 \chi_1[t_k] & \cdots & \Upsilon_L \chi_L[t_k] \end{bmatrix} \quad (16)$$

and

$$\varphi[t_k] = \left[\eta^{\frac{m}{2}} \mathbf{v}[t_{k-m}]^T, \dots, \eta^{\frac{0}{2}} \mathbf{v}[t_k]^T \right]^T \quad (17)$$

Then (15) can be rearranged as a classical linear regression problem (18):

$$\min_{\hat{\mathbf{x}}} \|\mathbf{Z}[t_k] \hat{\mathbf{x}} - \varphi[t_k]\|^2, \quad (18)$$

whose closed-form solution is:

$$\hat{\mathbf{x}} = \mathbf{Z}[t_k]^\dagger \varphi[t_k] \quad (19)$$

where $\mathbf{Z}[t_k]^\dagger = (\mathbf{Z}[t_k]^T \mathbf{Z}[t_k])^{-1} \mathbf{Z}[t_k]^T$ is the pseudo-inverse of $\mathbf{Z}[t_k]$. Then estimated vectors $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ can be calculated as:

$$\hat{\mathbf{A}} = (\mathbf{M}^{-1})^T \text{diag}(\text{diag}(\varsigma) \mathbf{Z}[t_k]^\dagger \varphi[t_k]) \mathbf{M}^{-1} \quad (20a)$$

$$\hat{\mathbf{B}} = (\mathbf{M}^{-1})^T \text{diag}(\mathbf{Z}[t_k]^\dagger \varphi[t_k]) \mathbf{M}^{-1} \quad (20b)$$

Notice that (20) is a closed-form formulation, related only to known parameters (e.g., network topology parameters and power injections) and a few measurements. Therefore, the proposed algorithm can be applied without incurring additional computational burden when there is no accurate network model. The analysis above shows that uncertainty of the feeder parameters is reflected in voltage measurements. Changes in feeder parameters inevitably cause changes in voltage measurements, which are exploited to identify true voltage sensitivity parameters in the optimization process to reflect changes in feeder parameters.

C. Proposed Algorithm Framework

A schematic diagram of the proposed algorithm framework is shown in Fig. 1. A dynamic system consisting of “prediction” and “correction” terms for distribution grids is developed to identify time-varying optimal power setpoints of DERs and network voltages, which automatically adapt to system disturbances, such as renewable energy sources, uncontrollable loads, and system model parameters.

In actual execution, output power increments of DERs are obtained based on predictive power parameters in ultra-short time and their gradients with respect to time, as well as voltage measurements. Setpoints of DERs are obtained by integrating

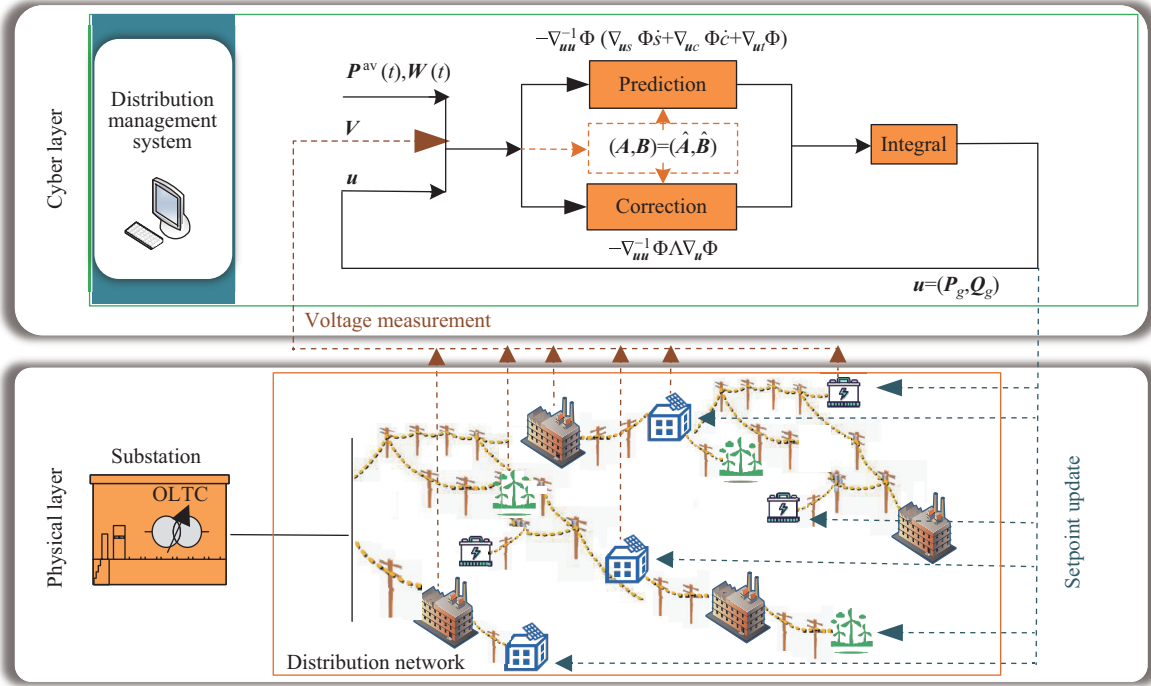


Fig. 1. Schematic diagram of the proposed time-varying tracking algorithm framework.

increments over time, which will affect system voltage through network constraints. After obtaining new power parameters and network voltages, new increments of DER setpoints can be calculated.

D. Convergence of Algorithm

Since sensitivity parameters are expressed as a closed solution form (20) in the estimation algorithm, we analyze convergence of the PC tracking method. Three assumptions that we impose on the optimization problem (3) are listed below.

Assumption 1: Constraint functions $\mathbf{u} \rightarrow f_i(\mathbf{u}, t), i \in [p]$ are convex and objective $\mathbf{u} \rightarrow f(\mathbf{u}, t)$ is uniformly strongly convex for all $t \geq 0$, i.e., $\nabla_{uu} f(\mathbf{u}, t) \geq m_f \mathbf{I}$ for some $m_f > 0$.

Assumption 2: Interior of the feasible region is nonempty for all $t \geq 0$.

After that, necessary and sufficient KKT conditions for optimality $\mathbf{u}^*(t)$ of (3) at each $t \geq 0$ can be given as

$$\nabla_u f(\mathbf{u}^*(t), t) + \sum_{i=1}^p \lambda_i^* \nabla_u f_i(\mathbf{u}^*(t), t) = 0 \quad (21a)$$

$$\lambda_i^* f_i(\mathbf{u}^*(t), t) = 0, \lambda_i^* \geq 0 \quad (21b)$$

$$f_i(\mathbf{u}^*(t), t) \leq 0, i \in [p] \quad (21c)$$

where $\lambda^* = [\lambda_1^* \dots \lambda_p^*] \in \Gamma^*(t)$.

According to Appendices C and D in [10], the following two lemmas hold, proof results which are followed and not detailed here.

Lemma 1: Define $\mathbf{u}^*(t)$ and $\hat{\mathbf{u}}^*(t)$ as the optimal solutions to (3) and (4), respectively. Then, under Assumptions 1 and 2, and for any $\lambda^* \in \Gamma^*(t)$ and $t \geq 0$, the following inequality holds:

$$|f(\hat{\mathbf{u}}^*(t), t) - f(\mathbf{u}^*(t), t)| \leq \frac{p}{c(t)} + s(t) \left(\inf_{\lambda^* \in \Gamma^*(t)} \|\lambda^*\|_1 \right) \quad (22)$$

Lemma 1 shows that if we choose $s(t)$ and $c(t)$ to make the right-hand side of (22) converge to zero, $\hat{\mathbf{u}}^*(t)$ will converge to $\mathbf{u}^*(t)$. There is one more assumption to rule out the possibility of optimal dual variables increasing exponentially to infinity:

Assumption 3: For any $\gamma > 0$, optimal dual variables satisfy $\lim_{t \rightarrow \infty} (\inf_{\lambda^* \in \Gamma^*(t)} \|\lambda^*\|_1) \exp(-\gamma t) = 0$, for all $i \in [p]$.

Lemma 2: Consider $\mathbf{u}(t)$ as the solution of (8) for $s(0) \geq \max_{i \in [p]} \{f_i(\mathbf{u}(0), 0)\}$ and $c(0) > 0$. Then, under Assumptions 1 and 2, $\mathbf{u}(t)$ satisfies $s(t) > f_i(\mathbf{u}, t), i \in [p]$ for all $t \geq 0$. Moreover,

$$\|\mathbf{u}^*(t) - \hat{\mathbf{u}}^*(t)\|_2 \leq C e^{-\alpha t} \quad (23)$$

where $0 \leq C := \frac{1}{m_f} \|\nabla_u \Phi(\mathbf{u}(0), c(0), s(0), 0)\|_2 < \infty$.

It can be derived from Assumption 3 and Lemma 1 that $\lim_{t \rightarrow \infty} |f(\hat{\mathbf{u}}^*(t), t) - f(\mathbf{u}^*(t), t)| = 0$ when $s(t) = s(0) \exp(-\gamma_s t)$, and $\lim_{t \rightarrow \infty} c(t) = \infty$. Because $f(\mathbf{u}, t)$ is strongly convex, we obtain $\lim_{t \rightarrow \infty} \|\hat{\mathbf{u}}^*(t) - \mathbf{u}^*(t)\|_2 = 0$. Combined with Lemma 2, we obtain that $\lim_{t \rightarrow \infty} \|\mathbf{u}(t) - \mathbf{u}^*(t)\|_2 = 0$.

IV. SIMULATION

Several numerical experiments on a modified IEEE 33-node distribution system [25] were carried out to evaluate the proposed method. The first experiment, under normal time-varying conditions, was performed to verify tracking performance of our method compared with several existing algorithms. The second experiment, with one PV halting and resuming operation suddenly, was presented to demonstrate

robustness of our approach under abnormal scenarios. The third experiment tested a network reconfiguration scenario to show benefits of the method's adaptivity to changes in system model. The dynamic optimization system illustrated in Fig. 1 was established in a MATLAB/Simulink platform. The computer was equipped with an Intel Core i7-4800MQ processor with 16 GB of RAM.

Figure 2 shows topology of the IEEE 33-node distribution system, where PVs were located at nodes {2, 6, 11, 16, 19, 24, 28}, WTs were added at nodes {13, 26}, and an ESS was located at node {31}. Available real power of DERs [26] and real power loads [27] are given in Fig. 3. Parameters of the ESS are $P_{\max}^{\text{ch}} = P_{\max}^{\text{dis}} = 200\text{kW}$, $\eta_c = \eta_d = 0.9$, $W_{\min} = 80\text{ kWh}$, and $W_{\max} = 320\text{ kWh}$. Let voltage limits $V_{\min i} = 0.95\text{ p.u.}$ and $V_{\max i} = 1.05\text{ p.u.}$, and $V_{\text{nom}} = 1\text{ p.u.}$ Other parameters for the proposed optimization problem are as follows: $C_p = 3$, $C_q = 1$, $\gamma = 1$, $\alpha = 100$, $m = 1$, $\eta = 1$, $s(t) = 2e^{-10t}$, $c(t) = e^{10t}$, and $\cos \theta = 0.85$.

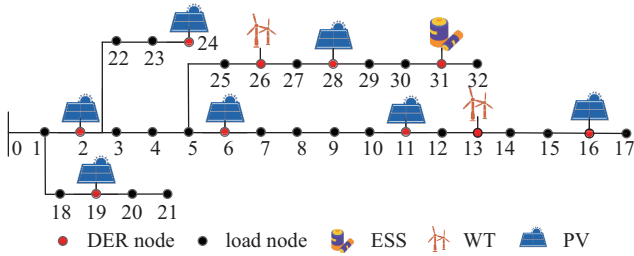


Fig. 2. IEEE 33-node distribution system topology.

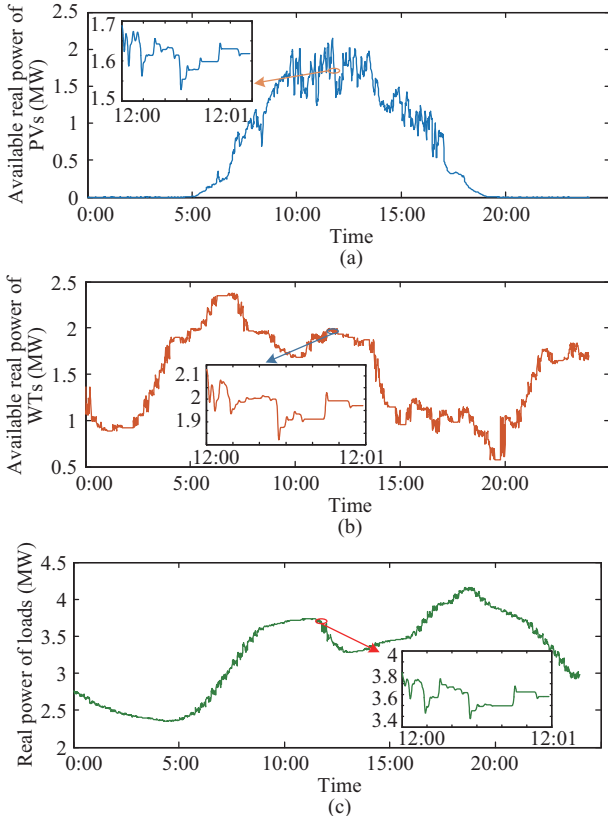


Fig. 3. Trajectories of DERs available power and load (a) Available real power of PVs. (b) Available real power of WTs. (c) Real power of loads.

A. Dynamic Trajectories Under Normal Time-Varying Conditions

First, a test under normal time-varying conditions was carried out from 12:00 p.m. to 12:01 p.m. A Gaussian distribution model $\varepsilon \sim N(0, \sigma^2)$ with $\sigma^2 = 0.001$ was added to rated line reactance parameters specified in [25] to simulate uncertain feeder parameters.

1) Results Comparison with Exact Optimal Solution

To verify efficiency and tracking performance of the proposed algorithm, exact optimal trajectories obtained by sampling the problem every 0.02 s and solved using the solver of the “yalmip” toolbox in MATLAB were used for comparison. Note here we assume the solver has sufficient time to obtain the optimal solution within each sampling interval, whose real computing time will be analyzed later. Fig. 4 shows trajectories of the power outputs of DERs and voltage magnitudes of some example nodes. Dashed lines represent track trajectories calculated by the proposed algorithm, while solid lines represent exact optimal trajectories.

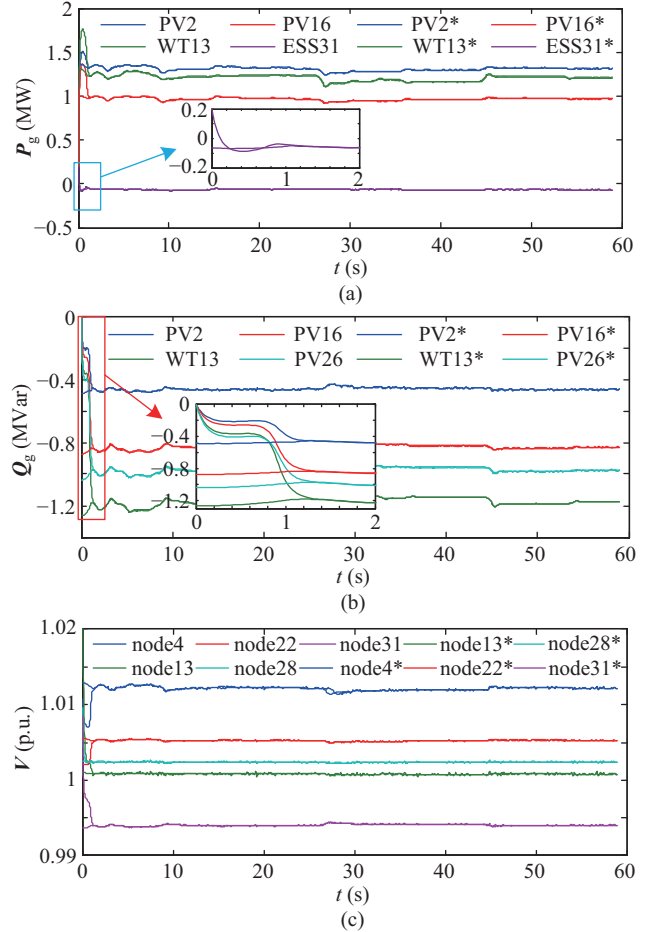


Fig. 4. Dynamic optimal trajectories of setpoints of DERs and voltage magnitudes under normal time-varying conditions. (a) Real power of DERs. (b) Reactive power of DERs. (c) Voltage magnitude.

As shown in Fig. 4, starting from initial values, trajectories of control variables (i.e., real and reactive power setpoints of DERs) and voltage magnitudes calculated by the proposed method gradually approached the optimal solution, tracked to

the exact optimal trajectories after about 1.5 s, and stayed on that optimum in the subsequent time-varying process. This means that as long as we start the algorithm, we will be able to identify the exact optimum within a short period of time (e.g., 1.5 s) compared to the entire tracking time, which can be set according to the specific situation, and in this case is set to be 60 s. The algorithm will remain on the optimal trajectory regardless of whether parameters change over time.

To demonstrate accuracy of our method, L_2 -norm of errors of control variables and the objective value vs. t were plotted in Fig. 5. As shown in Fig. 5, tracking errors drop rapidly and remained within a small range, such as 10^{-3} – 10^{-4} for control variables and 10^{-4} – 10^{-5} for objective value.

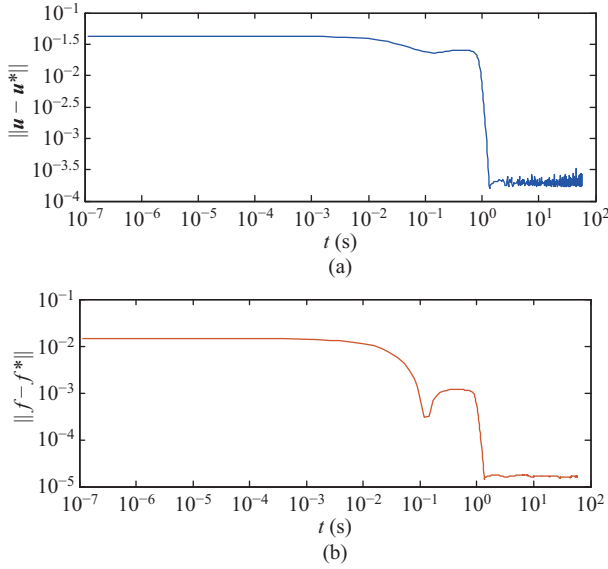


Fig. 5. Error trajectories of control variables and objective value. (a) Error trajectories of control variables. (b) Error trajectories of objective value.

2) Results Comparison with Existing Algorithms

To evaluate advantages of the proposed time-varying tracking algorithm, we compared the proposed algorithm with some existing algorithms, including the primal-dual algorithm [7] (without a “prediction” step), and discrete-time PC algorithm [15]. For existing algorithms, we sampled the time-varying optimization problem every 1 s, and iterated multiple times in the interval until convergence. Exact optimal trajectories obtained the same way as in 1) were used for comparison. Results of the real power setpoint of WT13 are shown in Fig. 6.

Analysis of tracking performance shows the trajectory obtained by the proposed algorithm (the green dotted line) can track exact optimal trajectory (the green solid line). In fact, the tracking error is so small that it is almost negligible. Result obtained by primal-dual algorithm (the orange solid line) always lags behind exact optimal trajectory, because it converges toward the optimal solution to the sampled problem, while the actual optimum is drifting with time, leading to a steady-state deviation. Discrete-time PC algorithm incorporates prediction of time-varying parameters so the result (the blue solid line) has little deviation at sampling points. However, it still cannot

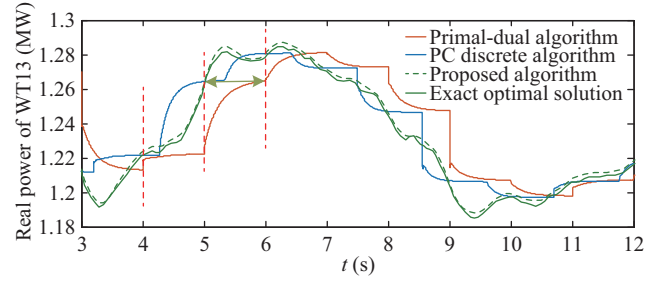


Fig. 6. Results comparison of existing algorithms and the proposed tracking algorithm.

fully capture dynamics in the interval, leading to a large error within the sampling interval.

We compare computation time of each algorithm in Table I.

TABLE I
COMPUTATION TIME COMPARISON OF DIFFERENT ALGORITHMS

Algorithms	Primal-dual	Discrete-time PC	Proposed
Time of each iteration (s)	3.95×10^{-4}	3.67×10^{-4}	0.0117
Time to get the optimal solution (s)	0.79	0.661	0.0117
	(with tracking deviation)		

As can be seen, it took the primal-dual and discrete-time PC algorithms 0.79 s and 0.661 s, respectively, to produce solutions with tracking errors. Although the proposed algorithm takes more time per iteration than the other two algorithms, it takes much less time to obtain the optimal solution (i.e., 0.0117s), which in practical applications is what matters to optimization algorithms. Therefore, the algorithm can be applied in real time at intervals of 0.02 s to match DER’s regulation time. According to our tests, real computing time for solver to solve the sampling problem is 0.419 s, which means that it is practically difficult for solver to obtain optimal solution at application intervals of 0.02 s. Here, we obtained the exact optimal solution for the sake of comparison only, without considering time cost.

3) Results Comparison with Fixed Sensitivity Method

To evaluate advantages of the sensitivity estimation method, in Fig. 7 we compared tracking errors between the proposed algorithm with estimated sensitivities and a simplified version of the proposed algorithm with fixed sensitivities (calculated from rate line parameters [25]). As shown, errors of both the control variables and objective value of the fixed sensitivity method are larger and oscillate more than of the proposed estimated sensitivity method.

B. Dynamic Trajectories When One PV Is Abnormal

To verify robustness of the method in abnormal scenarios, we assume PV28 halted operation at time $T = 12:00:25$ and resumed operation after 10 s. Test results are shown in Fig. 8.

When PV28 suddenly halts operation, other DERs begin to react and reach new optima at short notice. ESS deployed at node 31 was changed from the original charging state to discharge state to supplement power deficiency of the system (shown in Fig. 8(a)). When PV28 resumed operation, ESS31 returned to the charging state.

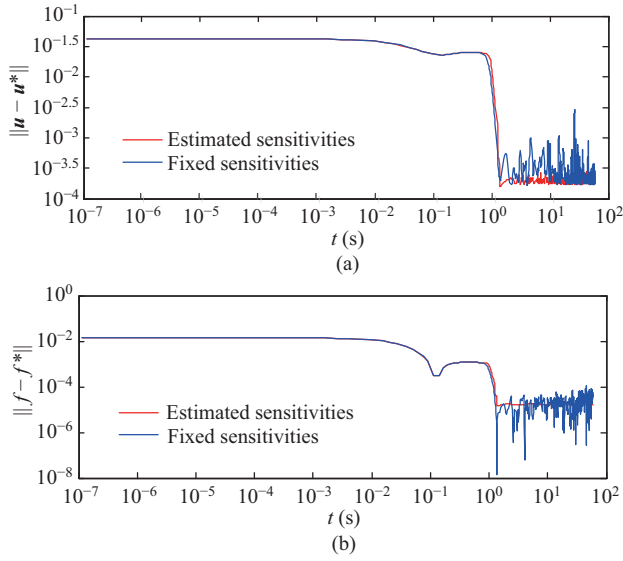


Fig. 7. Results comparison of methods with estimated and fixed sensitivities. (a) Error trajectories of control variables. (b) Error trajectories of objective value.

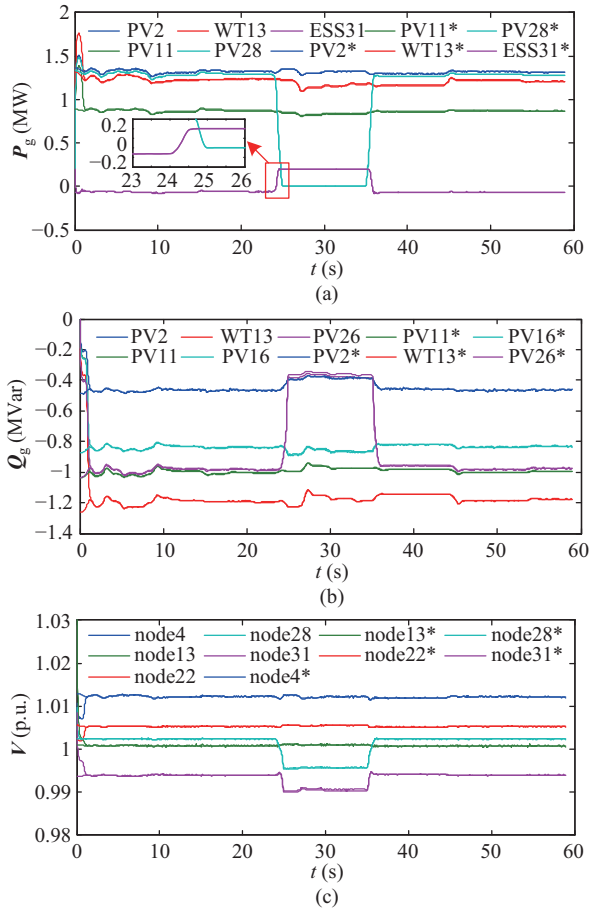


Fig. 8. Dynamic optimal trajectories of setpoints of DERs and voltage magnitudes when one PV is abnormal. (a) Real power of DERs. (b) Reactive power of DERs. (c) Voltage magnitude.

C. Dynamic Trajectories When Network is Reconfigured

A network reconfiguration scenario was tested at time $T = 12:00:30$, where bus 8 was connected to bus 29, and the line

between bus 5 and bus 25 was disconnected, as shown in Fig. 9.

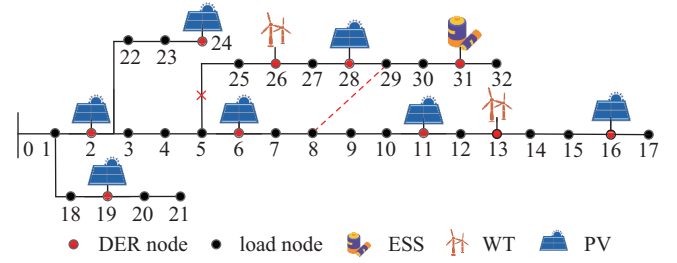


Fig. 9. IEEE 33-node distribution system reconfiguration topology.

Figure 10 shows power outputs of DERs and voltage magnitudes, from which we can see when the network structure suddenly changes, optimal trajectories also change suddenly. At this point, trajectories obtained by the proposed algorithm slightly deviate from optimal trajectories at the initial stage and return quickly to optimal trajectories, taking about 0.5 s.

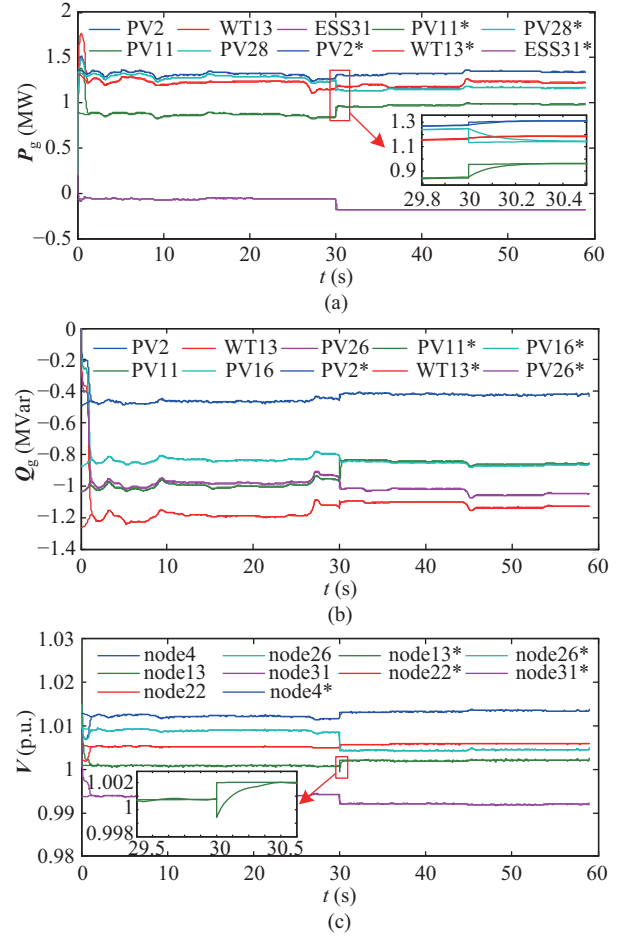


Fig. 10. Dynamic optimal trajectories of setpoints of DERs and voltage magnitudes during network reconfiguration. (a) Real power of DERs. (b) Reactive power of DERs. (c) Voltage magnitude.

V. CONCLUSION

In the present study, a novel tracking algorithm was designed to identify the optimal solution to a time-varying

optimization problem. It leverages time-varying information of the objective function and constraints to build a dynamic system, which demonstrates online tracking capability with an asymptotically vanishing error. Simulation results demonstrate the proposed algorithm has characteristics of fast-tracking speed and small tracking error. It also ensures more accurate tracking performance for time-varying problems than existing online algorithms and fixed sensitivity method. This study demonstrates the proposed method still has good tracking performance in case of fast-changing power and system parameters. Future research directions include extension of the proposed algorithm to consider scenarios in which DERs are owned by private sectors and prediction accuracy of power parameters is achieved in an ultra-short amount of time.

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